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BEHAVIOUR OF NUCLEON-SIXQUARK SYSTEM ON TEMPERATURE-DENSITY PLANE

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On the basis of the model for describing the heterogeneous states in nuclear matter the dependence of the behaviour of a two-phase system of nucleons (fermions) + sixquarks (bosons) is investigated at the temperature (θ) and density (ρ). Every phase is the collection of n-quark colourless clusters (n = 3 for nucleus and n = 6 for sixquark) described as the van-der-Waalse gas. Our analysis demonstrates the advantage of the heterogeneous state for a wide region of θ and ρ.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Поведение нуклон-шестикварковой системы на плоскости температура-плотность

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На основе модели для описания гетерофазных состояний в ядерной материи исследуется поведение двухфазной системы нуклоны (фермионы) + шестикварки (бозоны) в зависимости от температуры θ и плотности ρ. Каждая из фаз описывается как совокупность n-кварковых бесцветных кластеров (n = 3 для нуклона и n = 6 для шестикварка), взаимодействие между которыми учтено в духе ван-дер-Ваальса. Проведенный анализ показывает термодинамическую выгодность гетерофазного состояния в широком диапазоне значений θ и ρ.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

In our previous papers /1,2/ on the basis of the microscopic approach for describing the heterogeneous states /3,4/ we have suggested the model of a multiquark-cluster mixture in the nuclear matter. In this model matter is considered as a macroscopic system and we suppose that the nucleons, sixquark clusters and other multi-

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quark clusters are quasiparticle bound states of this matter, described in the framework of the theory of quark bags. For the simple case of the two-phase "nucleons + sixquarks" system at zero temperature it is shown that the heterogeneous state is more favourable compared with pure states for some values of density [1,2].

As far as the formulation of the model is general it is reasonable to consider nonzero temperatures. We shall investigate as well the dependence of our results on parametrization of the multiquark masses.

For the self-consistency of the paper we shall briefly describe the main principles of the model. According to the Bogolubov concept of quasiaverages [5] (see also [6]) the Hilbert space of states with definite symmetry properties corresponds to some thermodynamic phase. The representation of the Hamiltonian H_n is realized on the Hilbert space of states $\mathcal{H}_n$ with the corresponding symmetry properties. Then, the heterogeneous system is an equilibrium mixture of different phases with the Hamiltonian $H = \oplus_n H_n$ defined on the space [3,4] $\mathcal{H} = \oplus_n \mathcal{H}_n$. So, for the thermodynamic potential we have

$$\Omega = \sum_n \Omega_n, \quad \Omega_n = -\Theta \ln \underset{(\mathcal{H}_n)}{\mathrm{Sp}} e^{-H_n/\Theta},$$

where Θ is the temperature. The heterogeneous state is characterized by an additional order parameter [3,4]

$$W_n = \frac{N_n}{N}, \quad \sum_n W_n = 1,$$

where N is the total number of particles and N_n is the number of particles in the phase with the number n.

The multiquark states of the nuclear matter may be regarded as quasiparticle gas and the energy of each phase component is described by the Hamiltonian [1,2]

$$H_n = \sum_{\vec{k},s} (\epsilon_k^n - \mu_n)\, a_n^+(\vec{k}, s)\, a_n(\vec{k}, s). \tag{1}$$

Here $a_n^+(\vec{k}, s)$ is the quasiparticle creation operator for the quark cluster of n-type with the momentum $\vec{k}$ and the quantum number s meaning spin and isospin together, the energy spectrum being

$$\epsilon_k^n = \sqrt{\vec{k}^2 + M_n^2}, \tag{2}$$

where M_n is the mass of the quark cluster (bag) and μ_n is the corresponding chemical potential. It is obvious that for $n = 3$ the operator $a_n^+(a_n)$ is an operator of the Fermi-Dirac type but at $n = 6$ of the Bose-Einstein type [1,2]. We suppose that bags interact as hard spheres with the effective volume v_n each. Then the "free" volume for the motion of these bags is

$$V' = V - WN\frac{v_3}{3} - (1-W)N\frac{v_6}{6}, \tag{3}$$

where W is the concentration of the nucleon component. This corresponds to the van-der-Waalse gas /1,2/.

We consider first the behaviour of the system at low temperatures, when the nonrelativistic expansion of eq. (2) is possible. Then, taking into account the degeneracy factors for the nucleon ($g_3 = 4$) and for sixquark ($g_6 = 3$), for the free energy per one quark

$$f = \sum_n f_n, \; f_n = \Omega_n/N + \mu_n \tag{4}$$

we obtain

$$f = \frac{W}{3}\left[M_3 + \frac{3}{10M_3}\left(\frac{\pi^2}{2}\frac{W}{v'}\right)^{2/3} - \frac{M_3}{2}\left(2\pi\frac{v'}{W}\right)^{2/3}\Theta^2\right] + \frac{1-W}{6}M_6\left[1 - 1.5354\sqrt{M_6}\,\frac{v'}{1-W}\,\theta^{5/2}\right], \tag{5}$$

where $v' \equiv V'/N$. As is known, a stable state is the one with a minimal free energy $f(v, W, \Theta)$. Then, the phase concentration $W(v,\Theta)$ is defined by the condition of equilibrium $\partial f/\partial W = 0$. Thus, we get

$$M_3 - \frac{M_6}{2} + \frac{1}{5M_3}\left(\frac{\pi^2}{2}\frac{W}{v'}\right)^{2/3}\left[\frac{5}{2} - \frac{W}{v'}\left(\frac{v_6}{6} - \frac{v_3}{3}\right)\right] - \Theta^2\frac{M_3}{3}\left(2\pi\frac{v'}{W}\right)^{2/3} \times \left[\frac{1}{2} + \frac{W}{v'}\left(\frac{v_6}{6} - \frac{v_3}{3}\right)\right] - 0.7677\left(\frac{v_6}{6} - \frac{v_3}{3}\right)M_6^{3/2}\,\Theta^{5/2} = 0. \tag{6}$$

In the case of high temperatures, when the system becomes classic, but nonrelativistic expansion for eq.(6) is still correct, the specific free energy has the form

$$f(v, W, \Theta) = \frac{W}{3}\left\{M_3 - \Theta\left[\ln\left(12\left(\frac{M_3\Theta}{2\pi}\right)^{3/2}\frac{v'}{W}\right) + 1\right]\right\} + \frac{1-W}{6}\left\{M_6 - \Theta\left[\ln\left(18\left(\frac{M_6\Theta}{2\pi}\right)^{3/2}\frac{v'}{1-W}\right) + 1\right]\right\}, \tag{7}$$

and the equilibrium condition is given by the equation

$$\Theta\left\{\frac{1}{2}\ln\left[18\left(\frac{M_6\Theta}{2\pi}\right)^{3/2}\frac{v'}{1-W}\right] - \ln\left[12\left(\frac{M_3\Theta}{2\pi}\right)^{3/2}\frac{v'}{W}\right] - \left(\frac{v_6}{6} - \frac{v_3}{3}\right)\frac{1+W}{2v'}\right\} + M_3 - \frac{M_6}{2} = 0. \tag{8}$$

Note, that a solution of eq.(8) $W(v, \Theta)$ has a minimum at the temperature $\Theta_m = \frac{2}{3}(M_6 - 2M_3)$.

At asymptotically high temperatures $\Theta \gg M_{3,6}$ it is necessary to use the relativistic energy spectrum (2). Keeping the main terms with respect to Θ in eq.(4), for the specific free energy of the system we obtain

$$f(v, W, \Theta) = -\frac{W}{3}\Theta \ln\left(\frac{12}{\pi^2}\Theta^3\frac{v'}{W}\right) - \frac{1-W}{6}\Theta \ln\left(\frac{18}{\pi^2}\Theta^3\frac{v'}{1-W}\right), \quad (9)$$

while the equilibrium condition is

$$\frac{1}{2}\ln\left(\frac{18}{\pi^2}\Theta^3\frac{v'}{1-W}\right) - \ln\left(\frac{12}{\pi^2}\Theta^3\frac{v'}{W}\right) - \left(\frac{v_6}{6} - \frac{v_3}{3}\right)\frac{1+W}{2v'} = 0. \quad (10)$$

The analysis of eq.(10) gives the possibility of writing down the asymptotic form for the nucleon concentration

$$W \simeq 1 - \frac{\pi^2}{8}\,\frac{1}{v - \frac{v_3}{3}}\,\frac{1}{\Theta^3} \qquad (\Theta \to \infty). \quad (11)$$

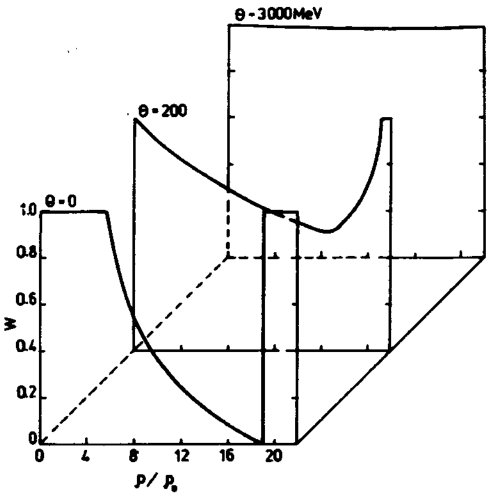

Fig.1

In Fig.1 the result of numerical calculation for W as a function of density $\rho = \frac{1}{v}$, or relative density ρ/ρ_0 (ρ_0 is the normal nuclear density $4\text{x}10^6$ MeV^3) for three typical temperatures is shown. The parameters of the model are taken as in papers [1,2].

At low temperatures (eq. (6)) the nucleation point, where the clusters of the sixquark phase appear, slightly increases with the temperature.

Let us emphasize that when choosing the value for the sixquark mass we could take it not from the bag theory as in [1,2] but from some experimental estimates [7-10], or fitting them by means of our formulae and the experimental data for the sixquark concentration [11-12].

The minimum of the nucleon concentration of temperature corresponds to $\Theta_m = 190$ MeV. At high temperatures and the normal nuclear density ($\rho = \rho_0$) there is a possibility for a coexistence of the nucleons and sixquarks ($W_6 \simeq 7\%$). In Fig.2 the advantage of the heterogeneous state "nucleons + sixquarks" at $\Theta =$ $= 200$ MeV is demonstrated for a wide region of densities.

In conclusion note the following fact. Eq. (11) at $\Theta = 0$ and $\rho = \rho_0$ permits one to define the minimal value of the sixquark (dibaryon) mass when the heterogeneous state is possible (see Fig.3). The result of calculations allows us to obtain an upper limit of the sixquark mass to be 1956 MeV, which is in a reasonable correspondence with experimental data [7]. Note, that the results are very sensitive to the value of the sixquark mass. For instance, if we take

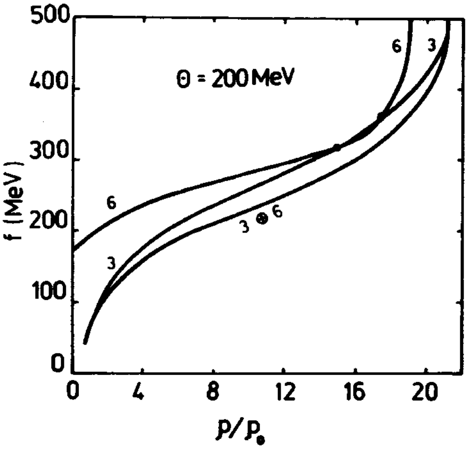

Fig.2

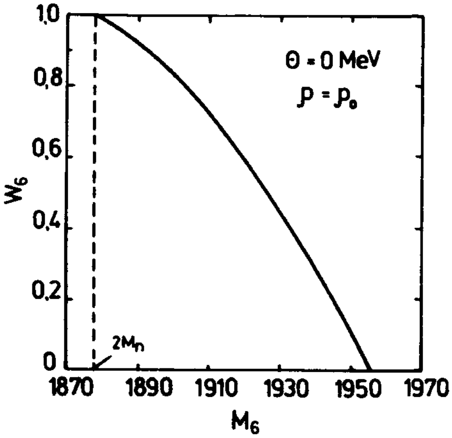

Fig.3

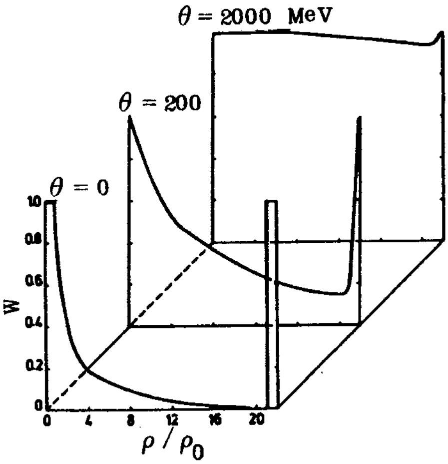

Fig4

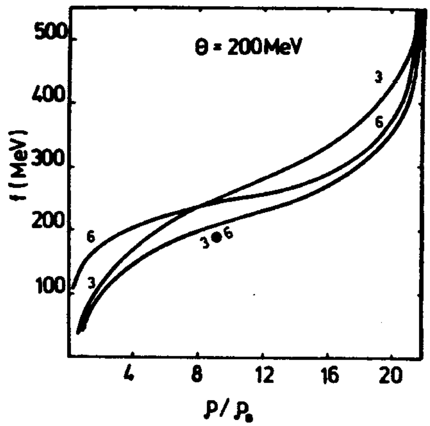

Fig.5

it be equal to 1950 MeV, then the sixquark concentration is about 10 %. In Fig.4 the behaviour of the nucleon concentration $W(\rho, \Theta)$ is shown for the case M_6 = 1950 MeV, and the corresponding free energies are shown in Fig.5.

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